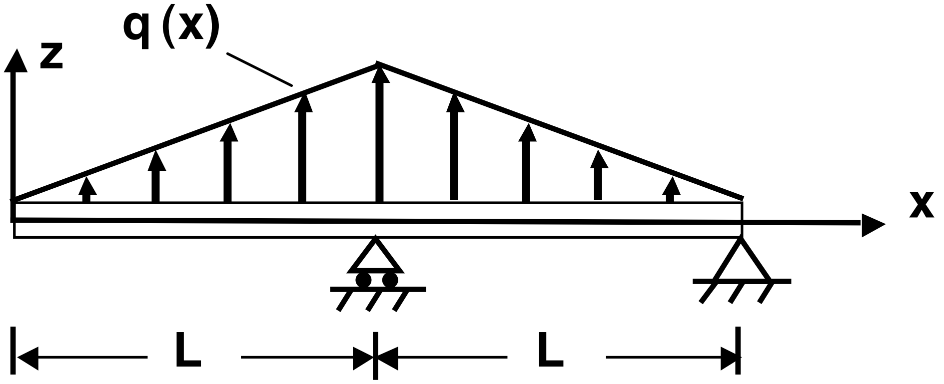
Unified Engineering Problem Set **Lectures: M4, M5, M6, M7 (part)**
Week 3 Spring, 2008 **Units: M4.3**

M3.1 ***(15 points)*** A beam of total length 2L is pinned at the far end and has a roller support at the mid-point. The beam is loaded by a linearly-varying distributed upward loading of magnitude q(x) that tapers to zero at each tip and reaches its peak at the mid-point. The integrated load due to the linearly varying distributed loading, q(x), is equal in magnitude to a value P.



(a) Determine the reactions for this structural configuration.

(b) Using the relationships between loading, shear, and moment, determine the loading, shear, and moment distributions. Draw these diagrams.

(c) Check the obtained values for these parameters at the midway point in each of the two beam bays (i.e., @ $x = L/2$, $x = 3L/2$).

M3.2 ***(15 points)*** Now that we have begun to learn about beams, we can use this to build on the simple models of airplanes that we worked on in Week 3 last term when we modeled the lift distribution on a wing. We first will further explore how wings carry load in level flight and consider, as our example, the new Boeing 787.

The wing of the airplane, as shown subsequently, can be modeled as a beam of total span L which has no supports. The beam has a concentrated load (the weight of the *fuselage*[1], its contents, and the *empennage*[2]) P at its center and a distributed load (the lift of the wing minus its weight) along its span. You learn from Fluids that this distribution is often modeled as varying, with different possible variations, along the span. We examined four possible variations in the Fall term problem:

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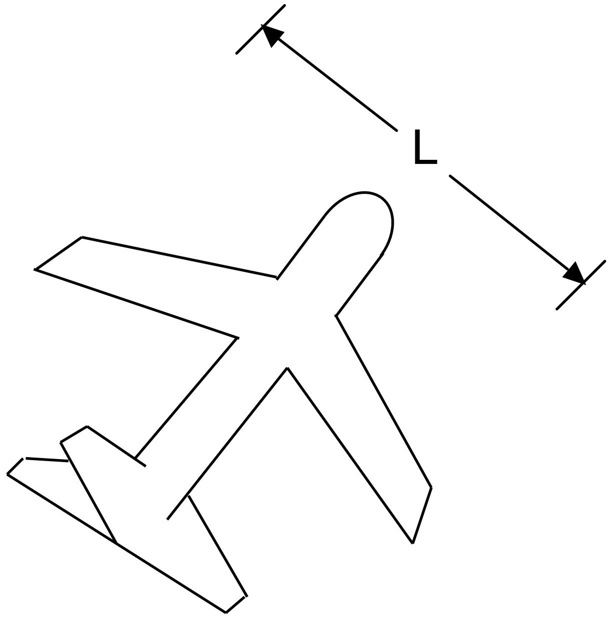
[1] The *fuselage* is that part of the airplane between the wings where the passengers and/or freight are carried.

[2] The *empennage* is more commonly known as the tail.



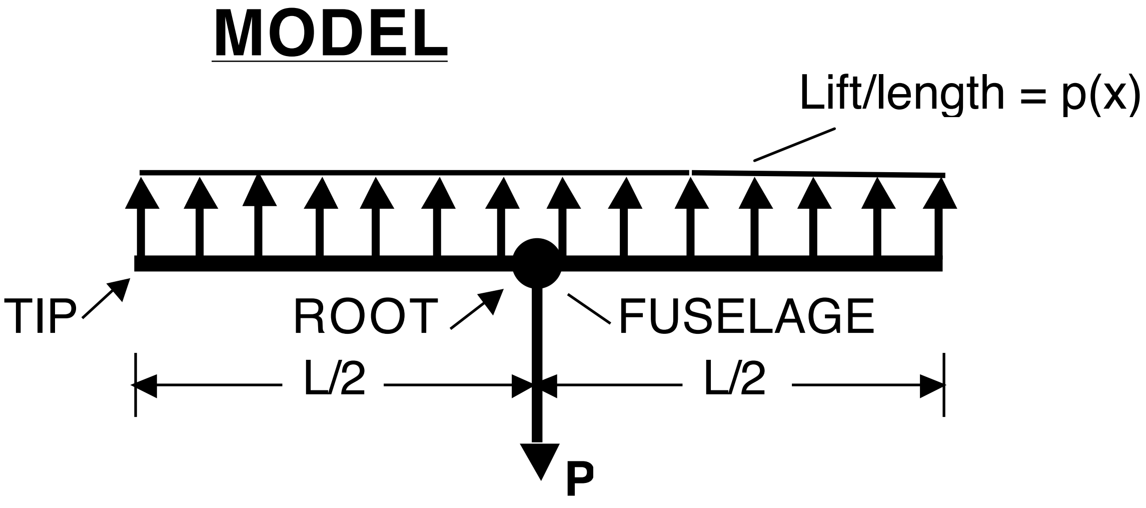
(1) constant along the *span* of the wing;

(2) linear variation along the *span* of the wing with the maximum value at the *root* and with a value of zero at the *tip*;

(3) linear variation along the *span* of the wing with maximum value at the *root* and with a value at the *tip* of half that at the *root*; and

(4) quadratic variation along the *span* of the wing (such that Lift = b - ax^2) with maximum value at the *root* and with a value of zero at the *tip*.

For simplicity, we ignore the weight of the wing in this work. The model is illustrated here for Case 1 (lift constant along the span).



For each case, perform analysis for parts (a) and (b). The specific (approximate) values for a 787 are a gross takeoff weight of 540,000 pounds, and a wing half-span of 100 feet.

(a) Determine the reactions for the structural configuration.

(b) Determine the axial force, shear force, and bending moment as functions of the distance from the *root*[3] of the wing.

Subsequently, compare the results for the various lift models:

(c) Directly compare the axial force, shear force, and bending moment as functions of distance from the *root* for the various models by plotting each of these on a common plot.

(d) Using common sense arguments and the results from part (b), describe where it is likely that the wing is most highly loaded for each case.

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[3] The *root* of the wing is the location where the wing is joined to the fuselage.